AP Key to Assignments

① Functions

1	E
2	D
3	E
4	C
5	B
6	A
7	D
8	C
9	C
10	C
11	A

② Limits

1	E
2	C
3	E
4	B
5	E
6	A
7	A
8	A
9	D
10	C
11	E
12	B
13	A
14	C
15	B

③ Derivative Theory

1	A
2	B
3	B
4	B
5	D
6	A
7	B
8	E
9	D
10	B
11	E
12	E
13	C
14	D
15	A
16	C
17	D
18	D
19	B
20	C
21	B
22	E
23	B

④ The First and Second Derivative

1	C
2	A
3	C
4	C
5	C
6	A
7	B
8	B
9	A
10	D
11	C
12	B
13	A
14	C
15	D
16	C
17	A
18	A
19	D
20	D
21	A
22	D
23	A
24	A
25	C
26	B
27	B
28	E
29	E
30	A
31	E
32	B
33	C
34	B
35	D

⑤ Derivative Applications

1	A
2	B
3	A
4	D
5	E
6	B
7	D
8	C
9	B
10	B
11	A
12	B
13	D
14	E
15	D
16	A
17	D
18	B
19	D

⑥ Integration Theory

1	D
2	B
3	C
4	D
5	A
6	B
7	C
8	B
9	D
10	C
11	C
12	B
13	B
14	D
15	C
16	E
17	E
18	E
19	C
20	D
21	A
22	E
23	E
24	D
25	E
26	D
27	C
28	D
29	D
30	E

⑦ Integration Applications

1	E
2	A
3	A
4	D
5	B
6	D
7	A
8	C
9	B
10	B
11	C
12	D
13	A
14	A
15	A
16	C
17	C
18	C
19	B
20	C
21	B

⑧ Diff. Equations

1	B
2	B
3	C
4	E
5	D
6	A
7	B
8	A
9	B
10	A
11	C
12	E
13	E

FUNCTIONS ASSIGNMENT

1. If $f(x) = e^{3\ln(x^2)}$, then $f'(x) =$

(A) $e^{3\ln(x^2)}$ (B) $\frac{3}{x^2} e^{3\ln(x^2)}$ (C) $6(\ln x)e^{3\ln(x^2)}$ (D) $5x^4$ (E) $6x^5$

2. What is the domain of the function f given by $f(x) = \frac{\sqrt{x^2 - 4}}{x - 3}$?

(A) $\{x: x \neq 3\}$ (B) $\{x: |x| \leqq 2\}$ (C) $\{x: |x| \geqq 2\}$
(D) $\{x: |x| \geqq 2 \text{ and } x \neq 3\}$ (E) $\{x: x \geqq 2 \text{ and } x \neq 3\}$

3. Which of the following functions has the fastest rate of growth as $x \to \infty$?

(A) $y = x^{18} - 5x$ (B) $y = 5x^2$ (C) $y = \ln x^2$ (D) $y = (\ln x)^2$ (E) $y = e^{0.01x}$

4. Let f and g be odd functions. If p, r, and s are nonzero functions defined as follows, which must be odd?

I. $p(x) = f(g(x))$
II. $r(x) = f(x) + g(x)$
III. $s(x) = f(x)g(x)$

(A) I only (B) II only (C) I and II only
(D) II and III only (E) I, II, and III

5. $4\cos\left(x + \frac{\pi}{3}\right) =$

(A) $2\sqrt{3}\cos x - 2\sin x$ (B) $2\cos x - 2\sqrt{3}\sin x$ (C) $2\cos x + 2\sqrt{3}\sin x$
(D) $2\sqrt{3}\cos x + 2\sin x$ (E) $4\cos x + 2$

6. Which of the following does NOT have a period of π?

(A) $f(x) = \sin\left(\frac{1}{2}x\right)$ (B) $f(x) = |\sin x|$ (C) $f(x) = \sin^2 x$
(D) $f(x) = \tan x$ (E) $f(x) = \tan^2 x$

7. If $f(x) = e^x \sin x$, then the number of zeros of f on the closed interval $[0, 2\pi]$ is

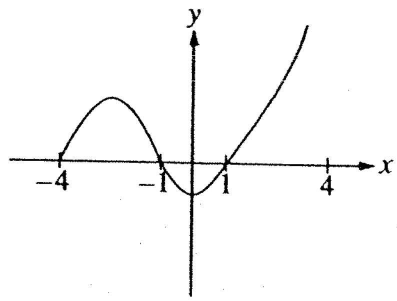
(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

8. If $\ln x - \ln\left(\frac{1}{x}\right) = 2$, then $x =$

(A) $\frac{1}{e^2}$ (B) $\frac{1}{e}$ (C) e (D) $2e$ (E) e^2

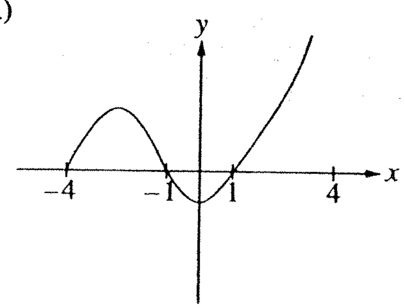
9. If $f(x) = \frac{x}{x+1}$, then the inverse function, f^{-1}, is given by $f^{-1}(x) =$

(A) $\frac{x-1}{x}$ (B) $\frac{x+1}{x}$ (C) $\frac{x}{1-x}$ (D) $\frac{x}{x+1}$ (E) x

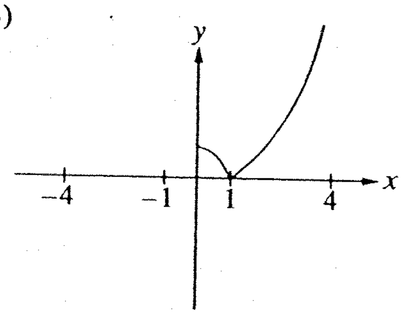


10. The graph of $y = f(x)$ is shown in the figure above. Which of the following could be the graph of $y = f(|x|)$?

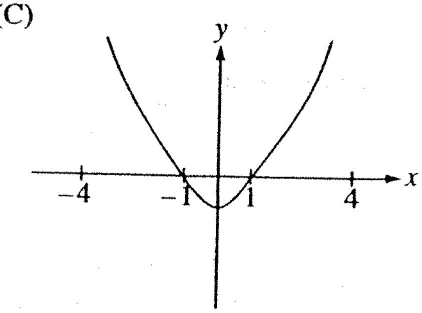
(A)



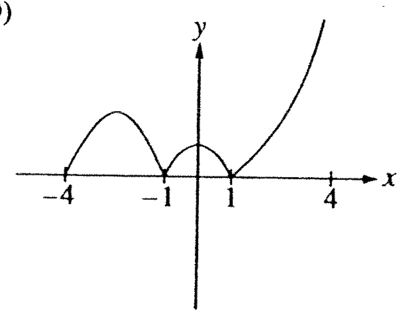
(B)



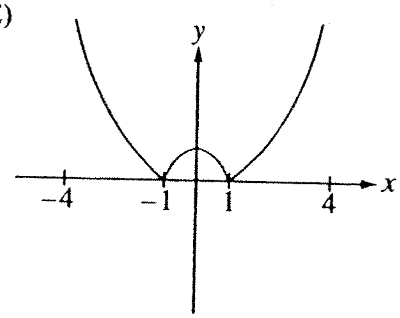
(C)



(D)



(E)



11. The fundamental period of $2\cos(3x)$ is

(A) $\frac{2\pi}{3}$ (B) 2π (C) 6π (D) 2 (E) 3

LIMITS ASSIGNMENT

1. $\lim_{x\to 1} \dfrac{x}{\ln x}$ is

(A) 0 (B) $\dfrac{1}{e}$ (C) 1 (D) e (E) nonexistent

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2. The graph of which of the following equations has $y = 1$ as an asymptote?

(A) $y = \ln x$ (B) $y = \sin x$ (C) $y = \dfrac{x}{x+1}$ (D) $y = \dfrac{x^2}{x-1}$ (E) $y = e^{-x}$

3. For $x \geq 0$, the horizontal line $y = 2$ is an asymptote for the graph of the function f. Which of the following statements must be true?

(A) $f(0) = 2$

(B) $f(x) \neq 2$ for all $x \geq 0$

(C) $f(2)$ is undefined.

(D) $\lim_{x\to 2} f(x) = \infty$

(E) $\lim_{x\to\infty} f(x) = 2$

---

4. Let $f(x) = \begin{cases} 3x^2 - 4, & \text{for } x \leq 1 \\ 6x - 5, & \text{for } x > 1 \end{cases}$

Which of the following are true statements about this function?

I. $\lim_{x\to 1} f(x)$ exists.

II. $\lim_{x\to 1} f'(x)$ exists.

III. $f'(1)$ exists.

(A) None (B) II only (C) III only (D) II and III (E) I, II, and III

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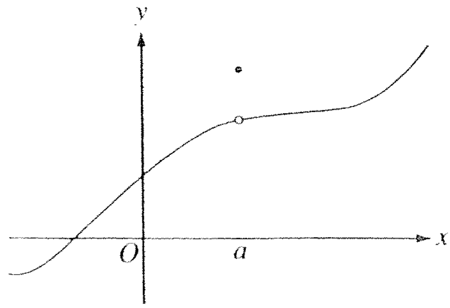
5. Let $f(x)$ be a continuous function that is defined for all real numbers x.
If $f(x) = \dfrac{x^2 - x - 6}{x^2 - 5x + 6}$ when $x^2 - 5x + 6 \neq 0$, what is $f(3)$?

(A) 0 (B) 1 (C) 2 (D) 4 (E) 5



6. The graph of a function f is shown above. Which of the following statements about f is false?

(A) f is continuous at $x = a$.

(B) f has a relative maximum at $x = a$.

(C) $x = a$ is in the domain of f.

(D) $\lim_{x \to a^+} f(x)$ is equal to $\lim_{x \to a^-} f(x)$.

(E) $\lim_{x \to a} f(x)$ exists.

---

7. Find $\lim_{x \to \infty} \dfrac{3x + 2x^3}{3x^3 - 4x^2 + 2x}$.

(A) $\frac{2}{3}$ (B) $\frac{3}{2}$ (C) 1 (D) $-\frac{1}{2}$ (E) $-\frac{3}{4}$

8. Let $g(x) = \lim_{h \to 0} \dfrac{(x+h)^2 - x^2}{h}$. For what value of x does $g(x) = 2$?

(A) $x = 1$ (B) $x = 2$ (C) $x = 3$ (D) $x = 4$ (E) $x = 5$

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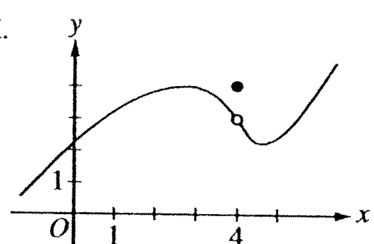
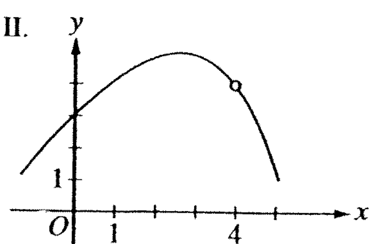
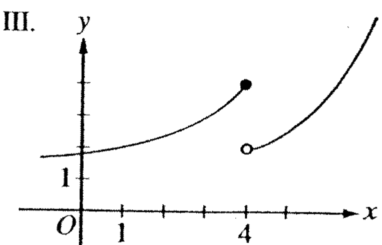
9. For which of the following does $\lim_{x \to 4} f(x)$ exist?

I. 

Graph of f

II. 

Graph of f

III. 

Graph of f

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I and III only

10. $\lim\limits_{\theta \to 0} \dfrac{1 - \cos\theta}{2\sin^2\theta}$ is

(A) 0 (B) $\frac{1}{8}$ (C) $\frac{1}{4}$ (D) 1 (E) nonexistent

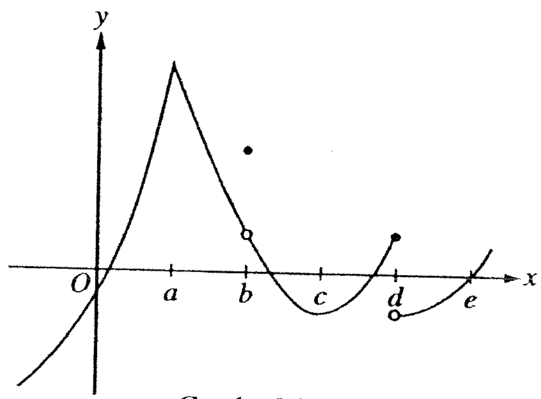
---

11. If the graph of $y = \dfrac{ax + b}{x + c}$ has a horizontal asymptote $y = 2$ and a vertical asymptote $x = -3$, then $a + c =$

(A) -5 (B) -1 (C) 0 (D) 1 (E) 5

12. The $\lim\limits_{h \to 0} \dfrac{\tan 3(x + h) - \tan(3x)}{h}$ is

(A) 0 (B) $3\sec^2(3x)$ (C) $\sec^2(3x)$ (D) $3\cot(3x)$ (E) nonexistent



Graph of f

13. The graph of a function f is shown above. At which value of x is f continuous, but not differentiable?

(A) a (B) b (C) c (D) d (E) e

14. Let f be a function such that $\lim\limits_{h \to 0} \dfrac{f(2 + h) - f(2)}{h} = 5$. Which of the following must be true?

I. f is continuous at $x = 2$.
II. f is differentiable at $x = 2$.
III. The derivative of f is continuous at $x = 2$.

(A) I only
(B) II only
(C) I and II only
(D) I and III only
(E) II and III only

15. If $a \neq 0$, then $\lim\limits_{x \to a} \dfrac{x^2 - a^2}{x^4 - a^4}$ is

(A) $\dfrac{1}{a^2}$ (B) $\dfrac{1}{2a^2}$ (C) $\dfrac{1}{6a^2}$ (D) 0 (E) nonexistent

APCD: Calculus

File Go Tools Help

16. The Problem Step by Step Scoring Guide Explore

For all real numbers x and y, let f be a function such that $f(x+y)=f(x)+f(y)+2xy$ and such that

$$\lim_{h\to 0}\frac{f(h)}{h}=7.$$

(a) Find $f(0)$. Justify your answer.

(b) Use the definition of the derivative to find $f'(x)$.

(c) Find $f(x)$.

Definition of the Derivative

Derivative Theory

1. If $\lim_{x \to 3} f(x) = 7$, which of the following must be true?

   I. f is continuous at $x = 3$.
   II. f is differentiable at $x = 3$.
   III. $f(3) = 7$

   (A) None
   (B) II only
   (C) III only
   (D) I and III only
   (E) I, II, and III

2. For all x in the closed interval $[2, 5]$, the function f has a positive first derivative and a negative second derivative. Which of the following could be a table of values for f?

(A)

x	$f(x)$
2	7
3	9
4	12
5	16

(B)

x	$f(x)$
2	7
3	11
4	14
5	16

(C)

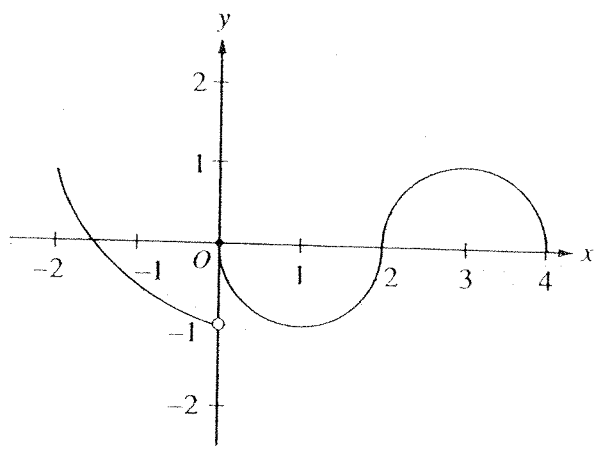
x	$f(x)$
2	16
3	12
4	9
5	7

(D)

x	$f(x)$
2	16
3	14
4	11
5	7

(E)

x	$f(x)$
2	16
3	13
4	10
5	7



3. The graph of the function f shown in the figure above has a vertical tangent at the point $(2, 0)$ and horizontal tangents at the points $(1, -1)$ and $(3, 1)$. For what values of x, $-2 < x < 4$, is f not differentiable?

   (A) 0 only (B) 0 and 2 only (C) 1 and 3 only (D) 0, 1, and 3 only (E) 0, 1, 2, and 3

4. If f is continuous for $a \le x \le b$ and differentiable for $a < x < b$, which of the following could be false?

   (A) $f'(c) = \dfrac{f(b) - f(a)}{b - a}$ for some c such that $a < c < b$.
   (B) $f'(c) = 0$ for some c such that $a < c < b$.
   (C) f has a minimum value on $a \le x \le b$.
   (D) f has a maximum value on $a \le x \le b$.
   (E) $\int_a^b f(x)\,dx$ exists.

5. Let $f(x) = g(h(x))$, where $h(2) = 3$, $h'(2) = 4$, $g(3) = 2$, and $g'(3) = 5$. Find $f'(2)$.

(A) 6

(B) 8

(C) 15

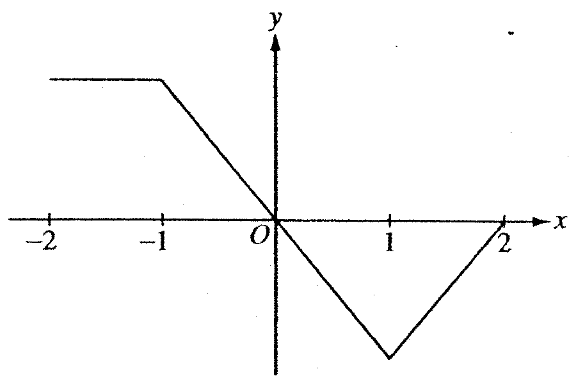
(D) 20

(E) More information is needed to find $f'(2)$.

x	0	1	2
$f(x)$	1	k	2

6. The function f is continuous on the closed interval $[0, 2]$ and has values that are given in the table above. The equation $f(x) = \frac{1}{2}$ must have at least two solutions in the interval $[0, 2]$ if $k =$

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) 3



Graph of f'

7. The graph of f', the derivative of the function f, is shown above. Which of the following statements is true about f ?

(A) f is decreasing for $-1 \le x \le 1$.

(B) f is increasing for $-2 \le x \le 0$.

(C) f is increasing for $1 \le x \le 2$.

(D) f has a local minimum at $x = 0$.

(E) f is not differentiable at $x = -1$ and $x = 1$.

8. Let g be a twice-differentiable function with $g'(x) > 0$ and $g''(x) > 0$ for all real numbers x, such that $g(4) = 12$ and $g(5) = 18$. Of the following, which is a possible value for $g(6)$?

(A) 15 (B) 18 (C) 21 (D) 24 (E) 27

9. Let f be a differentiable function with $f(2) = 3$ and $f'(2) = -5$, and let g be the function defined by $g(x) = xf(x)$. Which of the following is an equation of the line tangent to the graph of g at the point where $x = 2$?

(A) $y = 3x$

(B) $y - 3 = -5(x - 2)$

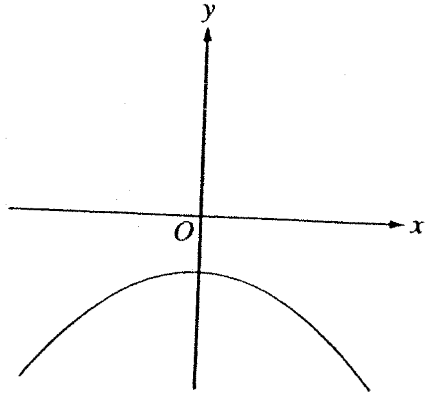
(C) $y - 6 = -5(x - 2)$

(D) $y - 6 = -7(x - 2)$

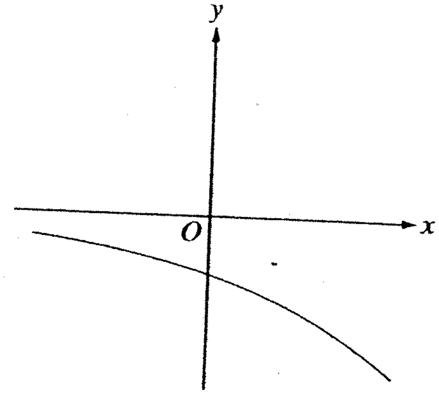
(E) $y - 6 = -10(x - 2)$

10. The function f has the property that $f(x)$, $f'(x)$, and $f''(x)$ are negative for all real values x. Which of the following could be the graph of f?

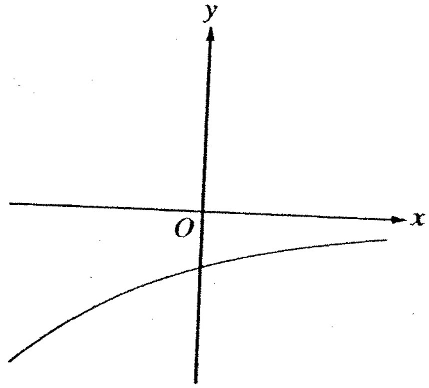
(A)



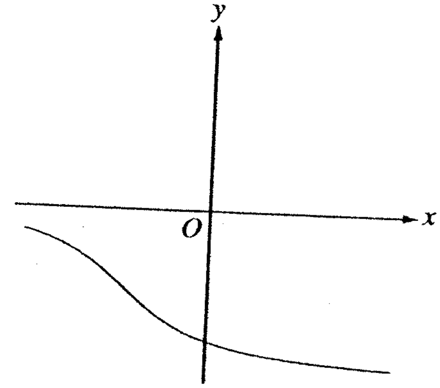
(B)



(C)



(D)



11. Let f be a function that is differentiable on the open interval $(1, 10)$. If $f(2) = -5$, $f(5) = 5$, and $f(9) = -5$, which of the following must be true?

I. f has at least 2 zeros.
II. The graph of f has at least one horizontal tangent.
III. For some c, $2 < c < 5$, $f(c) = 3$.

(A) None
(B) I only
(C) I and II only

12. Let f and g be differentiable functions with the following properties:

(i) $g(x) > 0$ for all x

(ii) $f(0) = 1$

If $h(x) = f(x)g(x)$ and $h'(x) = f(x)g'(x)$, then $f(x) =$

(A) $f'(x)$

(B) $g(x)$

(C) e^x

(D) 0

(E) 1

13. If f is a differentiable function, then $f'(a)$ is given by which of the following?

I. $\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

II. $\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

III. $\lim_{x \to a} \frac{f(x+h) - f(x)}{h}$

(A) I only (B) II only (C) I and II only (D) I and III only (E) I, II, and III

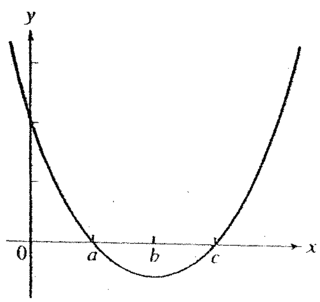
14.

x	3.3	3.4	3.5	3.6	3.7
$f(x)$	3.69	3.96	4.25	4.56	4.89

Let f be a differentiable function that is defined for all real numbers x. Use the table above to estimate $f'(3.5)$.

(A) 0.3 (B) 1.8 (C) 2.7 (D) 3.0 (E) 6.0

15.



The graph of $y = f'(x)$ is shown. Which of the following statements about the function $f(x)$ are true?

I. $f(x)$ is decreasing for all x between a and c.

II. The graph of f is concave up for all x between a and c.

III. $f(x)$ has a relative minimum at $x = a$.

(A) I only (B) II only (C) III only (D) I and III (E) I, II, and III

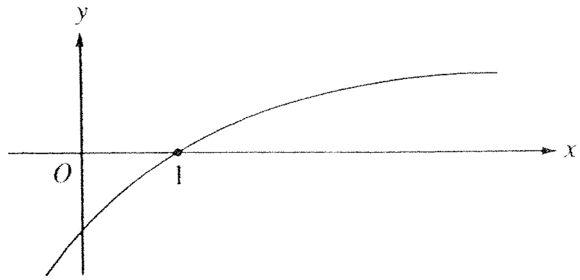
16. Let f be a twice-differentiable function whose derivative $f'(x)$ is increasing for all x. Which of the following must be true for all x?

I. $f(x) > 0$

II. $f'(x) > 0$

III. $f''(x) > 0$

(A) I only (B) II only (C) III only (D) I and II (E) II and III



17. The graph of a twice-differentiable function f is shown in the figure above. Which of the following is true?

(A) $f(1) < f'(1) < f''(1)$
(B) $f(1) < f''(1) < f'(1)$
(C) $f'(1) < f(1) < f''(1)$
(D) $f''(1) < f(1) < f'(1)$
(E) $f''(1) < f'(1) < f(1)$

18. If $f(x) = \sin\left(\frac{x}{2}\right)$, then there exists a number c in the interval $\frac{\pi}{2} < x < \frac{3\pi}{2}$ that satisfies the conclusion of the Mean Value Theorem. Which of the following could be c?

(A) $\frac{2\pi}{3}$ (B) $\frac{3\pi}{4}$ (C) $\frac{5\pi}{6}$ (D) π (E) $\frac{3\pi}{2}$

19. If g is a differentiable function such that $g(x) < 0$ for all real numbers x and if $f'(x) = (x^2 - 4)g(x)$, which of the following is true?

(A) f has a relative maximum at $x = -2$ and a relative minimum at $x = 2$.
(B) f has a relative minimum at $x = -2$ and a relative maximum at $x = 2$.
(C) f has relative minima at $x = -2$ and at $x = 2$.
(D) f has relative maxima at $x = -2$ and at $x = 2$.
(E) It cannot be determined if f has any relative extrema.

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20. Let f be a polynomial function with degree greater than 2. If $a \neq b$ and $f(a) = f(b) = 1$, which of the following must be true for at least one value of x between a and b?

I. $f(x) = 0$
II. $f'(x) = 0$
III. $f''(x) = 0$

(A) None (B) I only (C) II only (D) I and II only (E) I, II, and III

21. The function f is continuous for $-2 \le x \le 1$ and differentiable for $-2 < x < 1$. If $f(-2) = -5$ and $f(1) = 4$, which of the following statements could be false?

(A) There exists c, where $-2 < c < 1$, such that $f(c) = 0$.

(B) There exists c, where $-2 < c < 1$, such that $f'(c) = 0$.

(C) There exists c, where $-2 < c < 1$, such that $f(c) = 3$.

(D) There exists c, where $-2 < c < 1$, such that $f'(c) = 3$.

(E) There exists c, where $-2 \le c \le 1$, such that $f(c) \ge f(x)$ for all x on the closed interval $-2 \le x \le 1$.

22. At $x = 3$, the function given by $f(x) = \begin{cases} x^2, & x < 3 \\ 6x - 9, & x \geqq 3 \end{cases}$ is

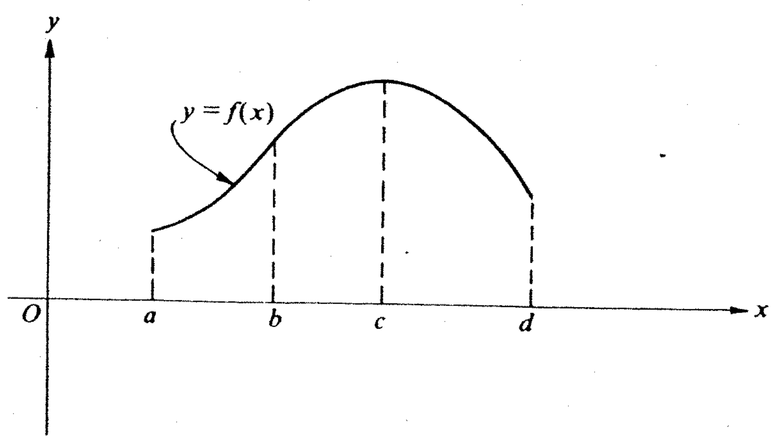
(A) undefined

(B) continuous but not differentiable

(C) differentiable but not continuous

(D) neither continuous nor differentiable

(E) both continuous and differentiable



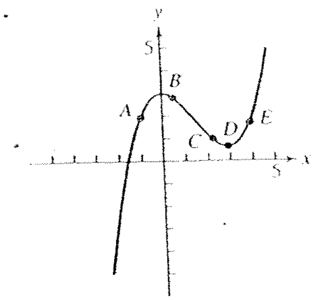
23. The graph of $y = f(x)$ is shown in the figure above. On which of the following intervals are $\frac{dy}{dx} > 0$ and $\frac{d^2y}{dx^2} < 0$?

I. $a < x < b$
II. $b < x < c$
III. $c < x < d$

(A) I only (B) II only (C) III only (D) I and II (E) II and III

The First and Second Derivative

1.



For the graph shown, at which point is it true that $\frac{dy}{dx} < 0$ and $\frac{d^2y}{dx^2} > 0$?

(A) A (B) B (C) C (D) D (E) E

2. If $f(x) = x\sqrt{2x-3}$, then $f'(x) =$

(A) $\frac{3x-3}{\sqrt{2x-3}}$ (D) $\frac{-x+3}{\sqrt{2x-3}}$

(B) $\frac{x}{\sqrt{2x-3}}$ (E) $\frac{5x-6}{2\sqrt{2x-3}}$

(C) $\frac{1}{\sqrt{2x-3}}$

3. How many critical points does the function $f(x) = (x+2)^5(x-3)^4$ have ?

(A) One (B) Two (C) Three (D) Five (E) Nine

4. What is the minimum value of $f(x) = x \ln x$?

(A) $-e$ (D) 0

(B) -1 (E) $f(x)$ has no minimum value.

(C) $-\frac{1}{e}$

5. If the derivative of f is given by $f'(x) = e^x - 3x^2$, at which of the following values of x does f have a relative maximum value?

(A) -0.46 (B) 0.20 (C) 0.91 (D) 0.95 (E) 3.73

6. If $x^2 + xy = 10$, then when $x = 2$, $\frac{dy}{dx} =$

(A) $-\frac{7}{2}$ (B) -2 (C) $\frac{2}{7}$ (D) $\frac{3}{2}$ (E) $\frac{7}{2}$

7. The function f has first derivative given by $f'(x) = \frac{\sqrt{x}}{1+x+x^3}$. What is the x-coordinate of the inflection point of the graph of f ?

(A) 1.008 (B) 0.473 (C) 0 (D) -0.278 (E) The graph of f has no inflection point.

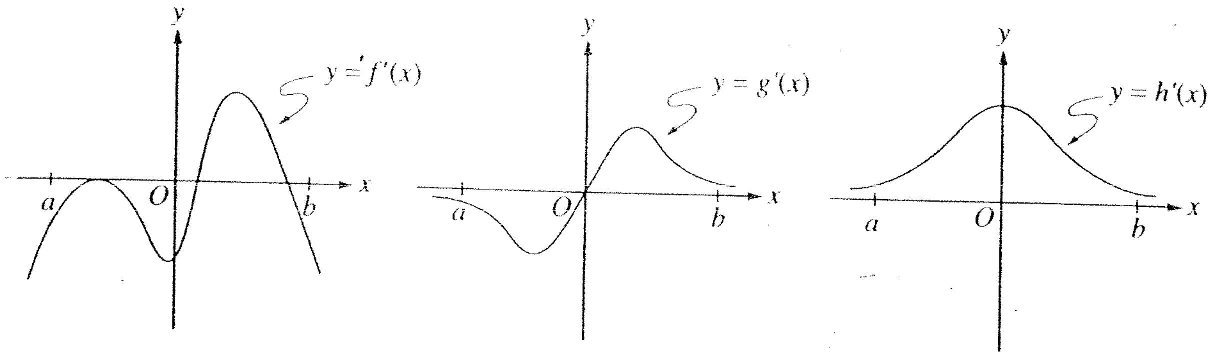
8. Let y be a differentiable function with $\frac{dy}{dx} > 0$ for all x. For which of the following values of y is it true that $\frac{d}{dx} y^2 = 8 \frac{d}{dx} \ln y$?

I. $y = \frac{1}{2}$

II. $y = 2$

III. $y = 4$

(A) I only (B) II only (C) III only (D) I and II (E) II and III



9. The graphs of the derivatives of the functions f, g, and h are shown above. Which of the functions f, g, or h have a relative maximum on the open interval $a < x < b$?

(A) f only
(B) g only
(C) h only
(D) f and g only
(E) f, g, and h

10. What is the instantaneous rate of change at $x = 2$ of the function f given by $f(x) = \frac{x^2 - 2}{x - 1}$?

(A) -2 (B) $\frac{1}{6}$ (C) $\frac{1}{2}$ (D) 2 (E) 6

11. At what value of x does the graph of $y = \frac{1}{x^2} - \frac{1}{x^3}$ have a point of inflection?

(A) 0 (B) 1 (C) 2 (D) 3 (E) At no value of x

12. Let $f(x) = \tan^{-1} x$. Find $f'(2)$.

(A) $\frac{\pi}{3}$ (B) $\frac{1}{5}$ (C) $\frac{1}{2}$ (D) $\frac{1}{2\sqrt{3}}$ (E) Undefined

13. Let f be a function defined for all real numbers x. If $f'(x) = \frac{|4 - x^2|}{x - 2}$, then f is decreasing on the interval

(A) $(-\infty, 2)$ (B) $(-\infty, \infty)$ (C) $(-2, 4)$ (D) $(-2, \infty)$ (E) $(2, \infty)$

14. Let f be a differentiable function such that $f(3) = 2$ and $f'(3) = 5$. If the tangent line to the graph of f at $x = 3$ is used to find an approximation to a zero of f, that approximation is

(A) 0.4 (B) 0.5 (C) 2.6 (D) 3.4 (E) 5.5

15. Let g be the function given by $g(x) = \int_0^x \sin(t^2)\,dt$ for $-1 \le x \le 3$. On which of the following intervals is g decreasing?

(A) $-1 \le x \le 0$

(B) $0 \le x \le 1.772$

(C) $1.253 \le x \le 2.171$

(D) $1.772 \le x \le 2.507$

(E) $2.802 \le x \le 3$

16. A particle moves along the x-axis so that at any time $t \ge 0$, its velocity is given by $v(t) = 3 + 4.1\cos(0.9t)$. What is the acceleration of the particle at time $t = 4$?

(A) -2.016 (B) -0.677 (C) 1.633 (D) 1.814 (E) 2.978

17. Find an equation of the line tangent to the graph of $y = \dfrac{3x}{x^2 - 6}$ at $x = 3$.

(A) $5x + y = 18$ (B) $5x - y = 12$ (C) $5x + 3y = 24$ (D) $x - 5y = -12$ (E) $x + y = 6$

18. If $f(x) = \ln\left(x + 4 + e^{-3x}\right)$, then $f'(0)$ is

(A) $-\frac{2}{5}$ (B) $\frac{1}{5}$ (C) $\frac{1}{4}$ (D) $\frac{2}{5}$ (E) nonexistent

19. If $f(x) = \ln\left|x^2 - 1\right|$, then $f'(x) =$

(A) $\left|\dfrac{2x}{x^2 - 1}\right|$

(B) $\dfrac{2x}{\left|x^2 - 1\right|}$

(C) $\dfrac{2\left|x\right|}{x^2 - 1}$

(D) $\dfrac{2x}{x^2 - 1}$

(E) $\dfrac{1}{x^2 - 1}$

20. Let $f(x) = x^3 - 12x$. Which statement about this function is false?

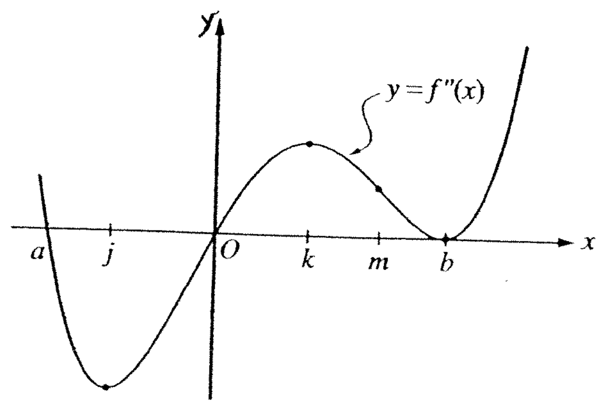
(A) The function has one inflection point.

(B) The function is concave upward for $x > 0$.

(C) The function has two relative extrema.

(D) The function is increasing for values of x between -2 and 2.

(E) The function has a relative minimum at $x = 2$.

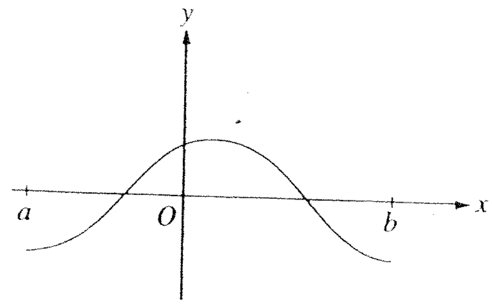


21. The second derivative of the function f is given by $f''(x) = x(x - a)(x - b)^2$. The graph of f'' is shown above. For what values of x does the graph of f have a point of inflection?

(A) 0 and a only (B) 0 and m only (C) b and j only (D) 0, a, and b (E) b, j, and k

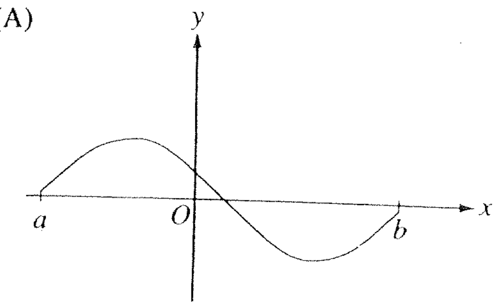
22. What are all values of x for which the function f defined by $f(x) = (x^2 - 3)e^{-x}$ is increasing?

(A) There are no such values of x.
(B) $x < -1$ and $x > 3$
(C) $-3 < x < 1$
(D) $-1 < x < 3$
(E) All values of x

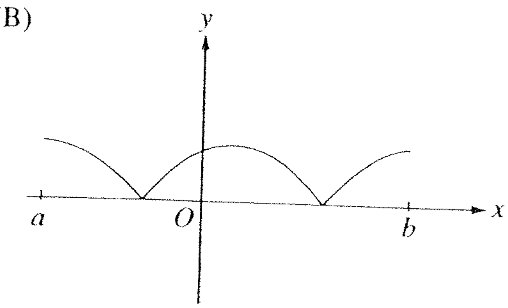


23. The graph of f is shown in the figure above. Which of the following could be the graph of the derivative of f?

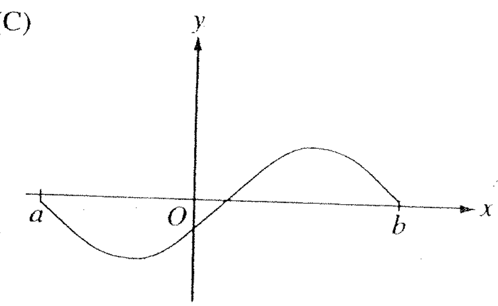
(A)



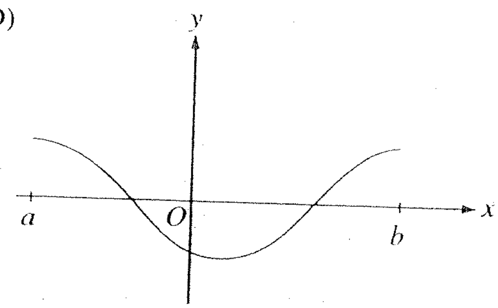
(B)



(C)



(D)



24. Let f be the function given by $f(x) = 2xe^x$. The graph of f is concave down when

(A) $x < -2$ (B) $x > -2$ (C) $x < -1$ (D) $x > -1$ (E) $x < 0$

25. $\frac{d}{dx}(2^x) =$

(A) 2^{x-1} (B) $(2^{x-1})x$ (C) $(2^x)\ln 2$ (D) $(2^{x-1})\ln 2$ (E) $\frac{2x}{\ln 2}$

26. What is the slope of the line tangent to the curve $3y^2 - 2x^2 = 6 - 2xy$ at the point $(3, 2)$?

(A) 0 (B) $\frac{4}{9}$ (C) $\frac{7}{9}$ (D) $\frac{6}{7}$ (E) $\frac{5}{3}$

27. Let f be the function defined by $f(x) = x^3 + x$. If $g(x) = f^{-1}(x)$ and $g(2) = 1$, what is the value of $g'(2)$?

(A) $\frac{1}{13}$ (B) $\frac{1}{4}$ (C) $\frac{7}{4}$ (D) 4 (E) 13

28. If $f(x) = \tan(2x)$, then $f'\left(\frac{\pi}{6}\right) =$

(A) $\sqrt{3}$ (B) $2\sqrt{3}$ (C) 4 (D) $4\sqrt{3}$ (E) 8

29. If $f(x) = \dfrac{e^{2x}}{2x}$, then $f'(x) =$

(A) 1

(B) $\dfrac{e^{2x}(1-2x)}{2x^2}$

(C) e^{2x}

(D) $\dfrac{e^{2x}(2x+1)}{x^2}$

(E) $\dfrac{e^{2x}(2x-1)}{2x^2}$

30. Let f be the function given by $f(x) = 2e^{4x^2}$. For what value of x is the slope of the line tangent to the graph of f at $(x, f(x))$ equal to 3 ?

(A) 0.168 (B) 0.276 (C) 0.318 (D) 0.342 (E) 0.551

31. Find the derivative of $\cos^3 2x$.

(A) $-\sin^3 2x$

(B) $6\cos^2 2x$

(C) $6\cos^2 2x \sin 2x$

(D) $-3\cos^2 2x \sin 2x$

(E) $-6\cos^2 2x \sin 2x$

32. Let $f(x) = x^5 + x$. Find the value of $\frac{d}{dx} f^{-1}(x)$ at $x = 2$.

(A) $-\dfrac{1}{6}$ (B) $\dfrac{1}{6}$ (C) $\dfrac{1}{81}$ (D) 6 (E) 81

33. Let f be the function given by $f(x) = 3e^{2x}$ and let g be the function given by $g(x) = 6x^3$. At what value of x do the graphs of f and g have parallel tangent lines?

(A) -0.701

(B) -0.567

(C) -0.391

34. Two particles are moving along the x-axis. Their positions are given by $x_1(t) = 2t^2 - 5t + 7$ and $x_2(t) = \sin 2t$, respectively. If $a_1(t)$ and $a_2(t)$ represent the acceleration functions of the particles, find the numbers of values of t in the closed interval [0, 5] for which $a_1(t) = a_2(t)$.

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4 or more

35. Which of the following is an equation of the line tangent to the graph of $f(x) = x^4 + 2x^2$ at the point where $f'(x) = 1$?

(A) $y = 8x - 5$

(B) $y = x + 7$

(C) $y = x + 0.763$

(D) $y = x - 0.122$

(E) $y = x - 2.146$

Derivative Applications

1. An equation of the line tangent to the graph of $f(x) = x(1 - 2x)^3$ at the point $(1, -1)$ is

(A) $y = -7x + 6$ (B) $y = -6x + 5$ (C) $y = -2x + 1$
(D) $y = 2x - 3$ (E) $y = 7x - 8$

---

2. At what point on the graph of $y = \frac{1}{2}x^2$ is the tangent line parallel to the line $2x - 4y = 3$?

(A) $\left(\frac{1}{2}, -\frac{1}{2}\right)$ (B) $\left(\frac{1}{2}, \frac{1}{8}\right)$ (C) $\left(1, -\frac{1}{4}\right)$ (D) $\left(1, \frac{1}{2}\right)$ (E) $(2, 2)$

3. The absolute maximum value of $f(x) = x^3 - 3x^2 + 12$ on the closed interval $[-2, 4]$ occurs at $x =$

(A) 4 (B) 2 (C) 1 (D) 0 (E) -2

4. The maximum acceleration attained on the interval $0 \le t \le 3$ by the particle whose velocity is given by $v(t) = t^3 - 3t^2 + 12t + 4$ is

(A) 9 (B) 12 (C) 14 (D) 21 (E) 40

---

5. A particle moves along the x-axis so that at time $t \ge 0$ its position is given by $x(t) = 2t^3 - 21t^2 + 72t - 53$. At what time t is the particle at rest?

(A) $t = 1$ only
(B) $t = 3$ only
(C) $t = \frac{7}{2}$ only
(D) $t = 3$ and $t = \frac{7}{2}$
(E) $t = 3$ and $t = 4$

6. The slope of the line normal to the graph of $y = 2 \ln(\sec x)$ at $x = \frac{\pi}{4}$ is

(A) -2
(B) $-\frac{1}{2}$
(C) $\frac{1}{2}$
(D) 2
(E) nonexistent

7. A particle moves along a line so that at time t, where $0 \le t \le \pi$, its position is given by $s(t) = -4\cos t - \frac{t^2}{2} + 10$. What is the velocity of the particle when its acceleration is zero?

(A) -5.19 (B) 0.74 (C) 1.32 (D) 2.55 (E) 8.13

8. The radius of a circle is increasing at a constant rate of 0.2 meters per second. What is the rate of increase in the area of the circle at the instant when the circumference of the circle is 20π meters?

(A) $0.04\pi\ \text{m}^2/\text{sec}$

(B) $0.4\pi\ \text{m}^2/\text{sec}$

(C) $4\pi\ \text{m}^2/\text{sec}$

(D) $20\pi\ \text{m}^2/\text{sec}$

(E) $100\pi\ \text{m}^2/\text{sec}$

---

9. A dog heading due north at a constant speed of 2 meters per second trots past a fire hydrant at $t = 0$ sec. Another dog heading due east at a constant speed of 3 meters per second trots by the hydrant at $t = 1$ sec. At $t = 9$ sec, the rate of change of the distance between the two dogs is

(A) 3.2 m/sec (B) 3.6 m/sec (C) 4.0 m/sec (D) 4.4 m/sec (E) 4.8 m/sec

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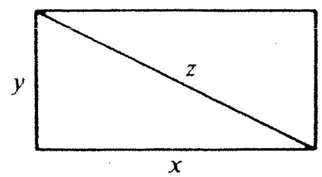


10. The sides of the rectangle above increase in such a way that $\frac{dz}{dt} = 1$ and $\frac{dx}{dt} = 3\frac{dy}{dt}$. At the instant when $x = 4$ and $y = 3$, what is the value of $\frac{dx}{dt}$?

(A) $\frac{1}{3}$ (B) 1 (C) 2 (D) $\sqrt{5}$ (E) 5

11. A railroad track and a road cross at right angles. An observer stands on the road 70 meters south of the crossing and watches an eastbound train traveling at 60 meters per second. At how many meters per second is the train moving away from the observer 4 seconds after it passes through the intersection?

(A) 57.60 (B) 57.88 (C) 59.20 (D) 60.00 (E) 67.40

12. If $y = 2x - 8$, what is the minimum value of the product xy?

(A) -16 (B) -8 (C) -4 (D) 0 (E) 2

---

13. Find the approximate value of x where $f(x) = x^2 - 3\sqrt{x+2}$ has its absolute minimum.

(A) -4.5 (B) -2 (C) 0 (D) 0.5 (E) 2.5

14. Let $f(x) = e^{x^3 - 2x^2 - 4x + 5}$. Then f has a local minimum at $x =$

(A) -2 (B) $-\frac{2}{3}$ (C) $\frac{2}{3}$ (D) 1 (E) 2

15. The volume of a cylindrical tin can with a top and a bottom is to be 16π cubic inches. If a minimum amount of tin is to be used to construct the can, what must be the height, in inches, of the can?

(A) $2\sqrt[3]{2}$ (B) $2\sqrt{2}$ (C) $2\sqrt[3]{4}$ (D) 4 (E) 8

16. Let $f(x) = \sqrt{x}$. If the rate of change of f at $x = c$ is twice its rate of change at $x = 1$, then $c =$

(A) $\frac{1}{4}$ (B) 1 (C) 4 (D) $\frac{1}{\sqrt{2}}$ (E) $\frac{1}{2\sqrt{2}}$

17. The top of a 25-foot ladder is sliding down a vertical wall at a constant rate of 3 feet per minute. When the top of the ladder is 7 feet from the ground, what is the rate of change of the distance between the bottom of the ladder and the wall?

(A) $-\frac{7}{8}$ feet per minute

(B) $-\frac{7}{24}$ feet per minute

(C) $\frac{7}{24}$ feet per minute

(D) $\frac{7}{8}$ feet per minute

(E) $\frac{21}{25}$ feet per minute

---

18. The radius of a circle is decreasing at a constant rate of 0.1 centimeter per second. In terms of the circumference C, what is the rate of change of the area of the circle, in square centimeters per second?

(A) $-(0.2)\pi C$

(B) $-(0.1)C$

(C) $-\frac{(0.1)C}{2\pi}$

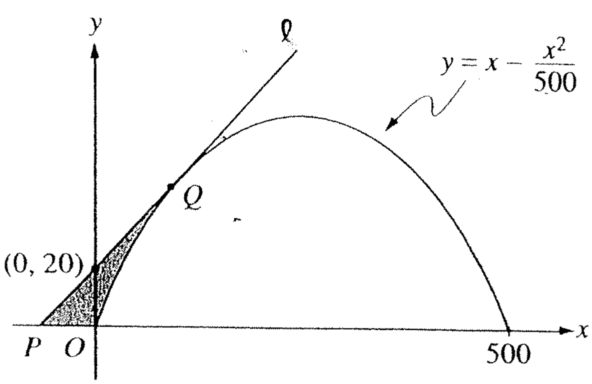
(D) $(0.1)^2 C$

---

19. The radius of a circle is increasing at a nonzero rate, and at a certain instant, the rate of increase in the area of the circle is numerically equal to the rate of increase in its circumference. At this instant, the radius of the circle is

(A) $\frac{1}{\pi}$ (B) $\frac{1}{2}$ (C) $\frac{2}{\pi}$ (D) 1 (E) 2

20.



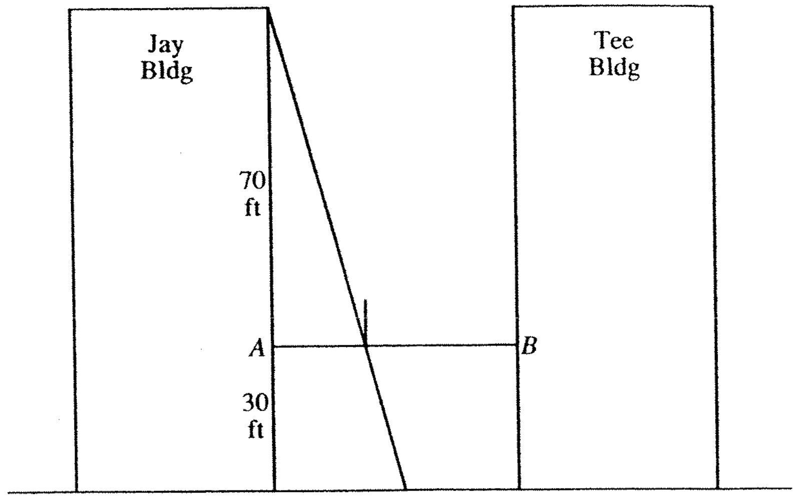
Line ℓ is tangent to the graph of $y = x - \frac{x^2}{500}$ at the point Q, as shown in the figure above.

(a) Find the x-coordinate of point Q.

(b) Write an equation for line ℓ.

(c) Suppose the graph of $y = x - \frac{x^2}{500}$ shown in the figure, where x and y are measured in feet, represents a hill. There is a 50-foot tree growing vertically at the top of the hill. Does a spotlight at point P directed along line ℓ shine on any part of the tree? Show the work that leads to your conclusion.

21.



A tightrope is stretched 30 feet above the ground between the Jay and the Tee buildings, which are 50 feet apart. A tightrope walker, walking at a constant rate of 2 feet per second from point A to point B, is illuminated by a spotlight 70 feet above point A, as shown in the diagram.

(a) How fast is the shadow of the tightrope walker's feet moving along the ground when she is midway between the buildings? (Indicate units of measure.)

(b) How far from point A is the tightrope walker when the shadow of her feet reaches the base of the Tee Building? (Indicate units of measure.)

(c) How fast is the shadow of the tightrope walker's feet moving up the wall of the Tee building when she is 10 feet from point B? (Indicate units of measure.)

22 Let h be a function defined for all $x \neq 0$ such that $h(4) = -3$ and the derivative of h is given by $h'(x) = \dfrac{x^2 - 2}{x}$ for all $x \neq 0$.

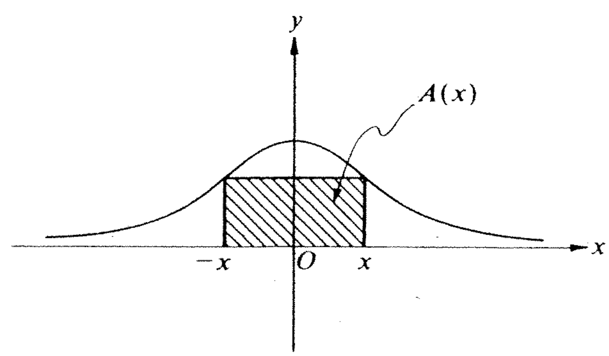
(a) Find all values of x for which the graph of h has a horizontal tangent, and determine whether h has a local maximum, a local minimum, or neither at each of these values. Justify your answers.

(b) On what intervals, if any, is the graph of h concave up? Justify your answer.

(c) Write an equation for the line tangent to the graph of h at $x = 4$.

(d) Does the line tangent to the graph of h at $x = 4$ lie above or below the graph of h for $x > 4$? Why?

23.



Let $A(x)$ be the area of the rectangle inscribed under the curve $y = e^{-2x^2}$ with vertices at $(-x, 0)$ and $(x, 0)$, $x \geq 0$, as shown in the figure above.

(a) Find $A(1)$.

(b) What is the greatest value of $A(x)$? Justify your answer.

(c) What is the average value of $A(x)$ on the interval $0 \leq x \leq 2$?

Integration Theory

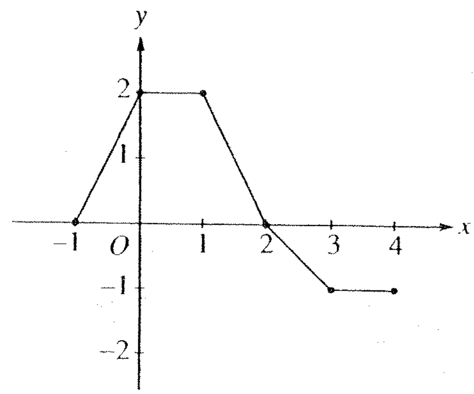
1. Suppose f and g are even functions that are continuous for all x, and let a be a real number. Which of the following expressions must have the same value?

   I. $\int_{-a}^{a} [f(x) + g(x)]\, dx$

   II. $2\int_{0}^{a} [f(x) + g(x)]\, dx$

   III. $\int_{-a}^{a} f(x)\, dx + \int_{-a}^{a} g(x)\, dx$

   (A) I and II only (B) I and III only (C) II and III only (D) I, II, and III (E) None



2. The graph of a piecewise-linear function f, for $-1 \le x \le 4$, is shown above. What is the value of $\int_{-1}^{4} f(x)\, dx$?

   (A) 1 (B) 2.5 (C) 4 (D) 5.5 (E) 8

3. If $\int_{a}^{b} f(x)\, dx = a + 2b$, then $\int_{a}^{b} (f(x) + 5)\, dx =$

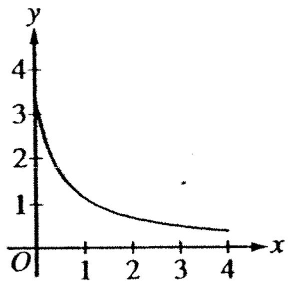
   (A) $a + 2b + 5$ (B) $5b - 5a$ (C) $7b - 4a$ (D) $7b - 5a$ (E) $7b - 6a$

4. If the definite integral $\int_{0}^{2} e^{x^2}\, dx$ is first approximated by using two <u>inscribed</u> rectangles of equal width and then approximated by using the trapezoidal rule with $n = 2$, the difference between the two approximations is

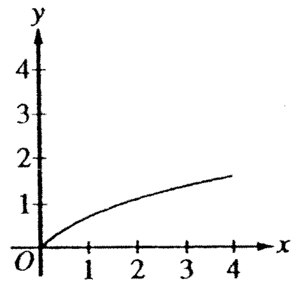
   (A) 53.60 (B) 30.51 (C) 27.80 (D) 26.80 (E) 12.78

5. If a trapezoidal sum overapproximates $\int_0^4 f(x)\,dx$, and a right Riemann sum underapproximates $\int_0^4 f(x)\,dx$, which of the following could be the graph of $y = f(x)$?

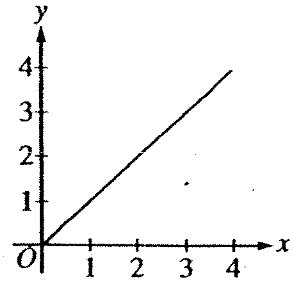
(A)



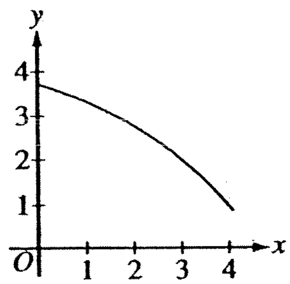
(B)



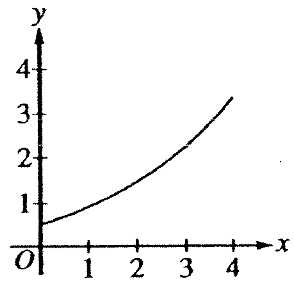
(E)



(C)



(D)



6. The expression $\frac{1}{50}\left(\sqrt{\frac{1}{50}} + \sqrt{\frac{2}{50}} + \sqrt{\frac{3}{50}} + \cdots + \sqrt{\frac{50}{50}}\right)$ is a Riemann sum approximation for

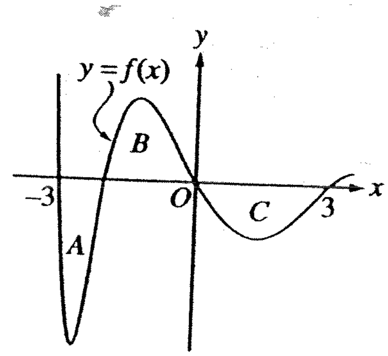
(A) $\int_0^1 \sqrt{\frac{x}{50}}\,dx$

(B) $\int_0^1 \sqrt{x}\,dx$

(C) $\frac{1}{50}\int_0^1 \sqrt{\frac{x}{50}}\,dx$

(D) $\frac{1}{50}\int_0^1 \sqrt{x}\,dx$

(E) $\frac{1}{50}\int_0^{50} \sqrt{x}\,dx$

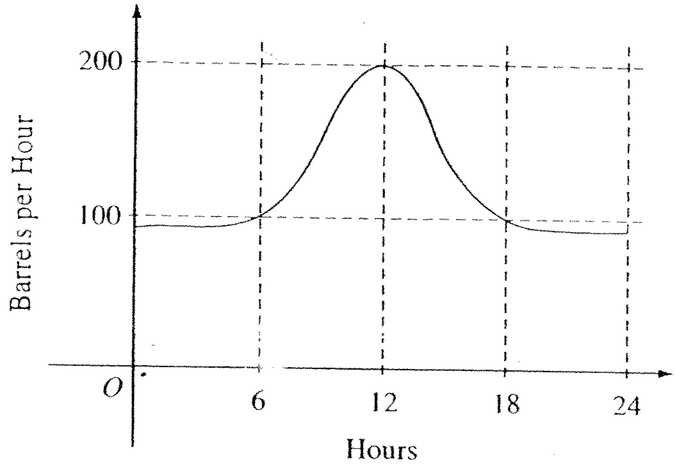


7. The regions A, B, and C in the figure above are bounded by the graph of the function f and the x-axis. If the area of each region is 2, what is the value of $\int_{-3}^{3} (f(x) + 1)\,dx$?

(A) -2 (B) -1 (C) 4 (D) 7 (E) 12

8. $\int_1^{500} (13^x - 11^x)\,dx + \int_2^{500} (11^x - 13^x)\,dx =$

(A) 0.000 (B) 14.946 (C) 34.415 (D) 46.000 (E) 136.364



9. The flow of oil, in barrels per hour, through a pipeline on July 9 is given by the graph shown above. Of the following, which best approximates the total number of barrels of oil that passed through the pipeline that day?

(A) 500 (B) 600 (C) 2,400 (D) 3,000 (E) 4,800

---

10. If $\int_0^k (2kx - x^2)dx = 18$, then $k =$

(A) -9 (B) -3 (C) 3 (D) 9 (E) 18

11. Using the substitution $u = 2x + 1$, $\int_0^2 \sqrt{2x+1}\,dx$ is equivalent to

(A) $\frac{1}{2}\int_{-1/2}^{1/2} \sqrt{u}\,du$ (B) $\frac{1}{2}\int_0^2 \sqrt{u}\,du$ (C) $\frac{1}{2}\int_1^5 \sqrt{u}\,du$ (D) $\int_0^2 \sqrt{u}\,du$ (E) $\int_1^5 \sqrt{u}\,du$

12. If f and g are continuous functions, and if $f(x) \geq 0$ for all real numbers x, which of the following must be true?

I. $\int_a^b f(x)g(x)\,dx = \left(\int_a^b f(x)\,dx\right)\left(\int_a^b g(x)\,dx\right)$

II. $\int_a^b \left(f(x) + g(x)\right)\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$

III. $\int_a^b \sqrt{f(x)}\,dx = \sqrt{\int_a^b f(x)\,dx}$

(A) I only (B) II only (C) III only (D) II and III only (E) I, II, and III

13. Evaluate $\int(\cos x - e^{2x})\,dx$.

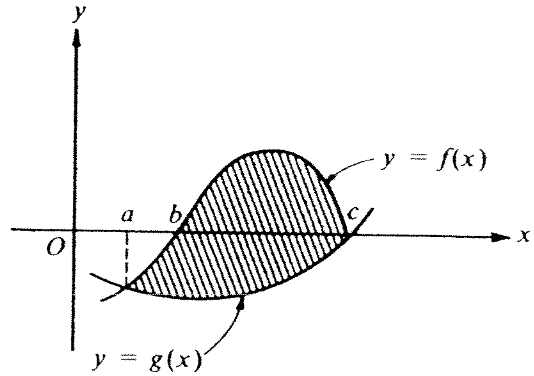
(A) $-\sin x - \frac{1}{2}e^{2x} + C$

(B) $\sin x - \frac{1}{2}e^{2x} + C$

(C) $-\sin x - 2e^{2x} + C$

(D) $\sin x - 2e^{2x} + C$

(E) $-\cos x - \frac{1}{2}e^{2x} + C$



14. The area of the shaded region in the figure above is represented by which of the following integrals?

(A) $\int_a^c (|f(x)| - |g(x)|)dx$

(B) $\int_b^c f(x)dx - \int_a^c g(x)dx$

(C) $\int_a^c (g(x) - f(x))dx$

(D) $\int_a^c (f(x) - g(x))dx$

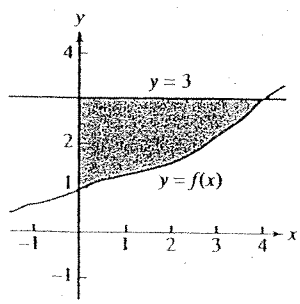
(E) $\int_a^b (g(x) - f(x))dx + \int_b^c (f(x) - g(x))dx$

x	2	5	7	8
$f(x)$	10	30	40	20

15. The function f is continuous on the closed interval [2, 8] and has values that are given in the table above. Using the subintervals [2, 5], [5, 7], and [7, 8], what is the trapezoidal approximation of $\int_2^8 f(x)dx$?

(A) 110 (B) 130 (C) 160 (D) 190 (E) 210

16.



Assume that $f(x)$ is a one-to-one function. The area of the shaded region is equal to which of the following definite integrals?

I. $\int_0^4 [f(x) - 3]\, dx$

II. $\int_4^0 [f(x) - 3]\, dx$

III. $\int_1^3 f^{-1}(y)\, dy$

(A) I only (B) II only (C) III only (D) I and III (E) II and III

17. $\int (x^2 + 1)^2 \, dx =$

(A) $\dfrac{(x^2+1)^3}{3} + C$

(B) $\dfrac{(x^2+1)^3}{6x} + C$

(C) $\left(\dfrac{x^3}{3} + x\right)^2 + C$

(D) $\dfrac{2x(x^2+1)^3}{3} + C$

(E) $\dfrac{x^5}{5} + \dfrac{2x^3}{3} + x + C$

18. For $x > 0$, $\int \left(\frac{1}{x} \int_1^x \frac{du}{u} \right) dx =$

(A) $\dfrac{1}{x^3} + C$

(B) $\dfrac{8}{x^4} - \dfrac{2}{x^2} + C$

(C) $\ln(\ln x) + C$

(D) $\dfrac{\ln(x^2)}{2} + C$

(E) $\dfrac{(\ln x)^2}{2} + C$

19. Let $F(x)$ be an antiderivative of $\dfrac{(\ln x)^3}{x}$. If $F(1) = 0$, then $F(9) =$

(A) 0.048 (B) 0.144 (C) 5.827 (D) 23.308 (E) 1,640.250

20. Which of the following are antiderivatives of $f(x) = \sin x \cos x$?

I. $F(x) = \dfrac{\sin^2 x}{2}$

II. $F(x) = \dfrac{\cos^2 x}{2}$

III. $F(x) = \dfrac{-\cos(2x)}{4}$

(A) I only
(B) II only
(C) III only
(D) I and III only
(E) II and III only

21. If f is a linear function and $0 < a < b$, then $\int_a^b f''(x)\,dx =$

(A) 0 (B) 1 (C) $\dfrac{ab}{2}$ (D) $b - a$ (E) $\dfrac{b^2 - a^2}{2}$

22. An antiderivative for $\dfrac{1}{x^2 - 2x + 2}$ is

(A) $-(x^2 - 2x + 2)^{-2}$

(B) $\ln(x^2 - 2x + 2)$

(C) $\ln\left|\dfrac{x-2}{x+1}\right|$

(D) $\text{Arcsec}\,(x - 1)$

(E) $\text{Arctan}\,(x - 1)$

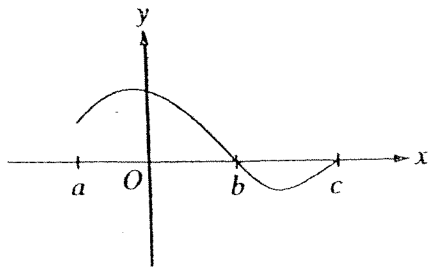
23. $\dfrac{d}{dx}\left(\displaystyle\int_0^{x^2} \sin\left(t^3\right) dt\right) =$

(A) $-\cos\left(x^6\right)$ (B) $\sin\left(x^3\right)$ (C) $\sin\left(x^6\right)$ (D) $2x\sin\left(x^3\right)$ (E) $2x\sin\left(x^6\right)$

24. $\displaystyle\int \frac{x\,dx}{\sqrt{3x^2+5}} =$

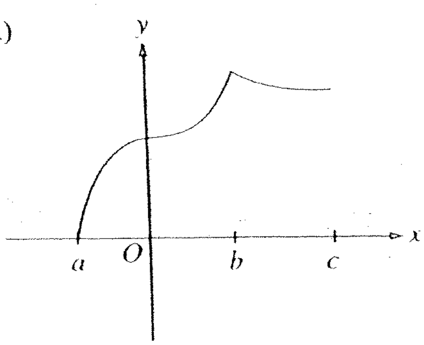
(A) $\frac{1}{9}(3x^2+5)^{\frac{3}{2}} + C$ (B) $\frac{1}{4}(3x^2+5)^{\frac{3}{2}} + C$ (C) $\frac{1}{12}(3x^2+5)^{\frac{1}{2}} + C$

(D) $\frac{1}{3}(3x^2+5)^{\frac{1}{2}} + C$ (E) $\frac{3}{2}(3x^2+5)^{\frac{1}{2}} + C$

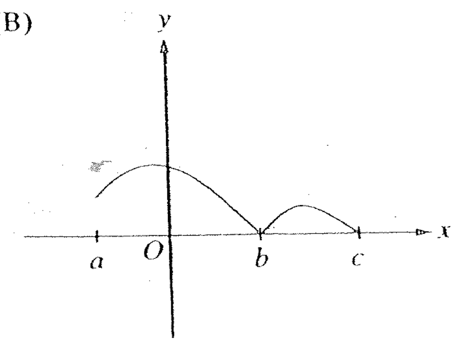


25. Let $f(x) = \displaystyle\int_a^x h(t)\,dt$, where h has the graph shown above. Which of the following could be the graph of f?

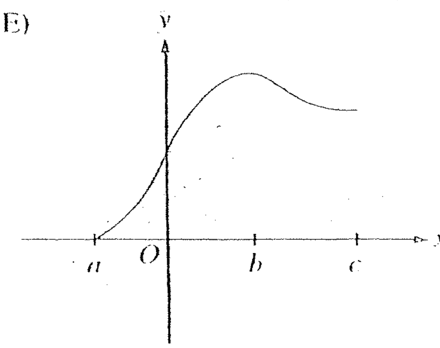
(A)



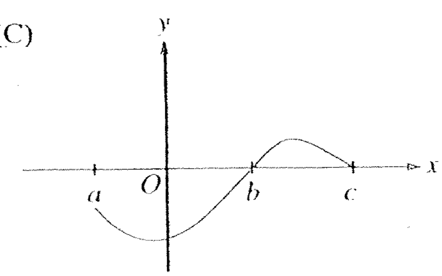
(B)



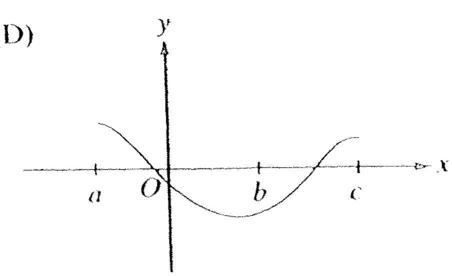
(E)



(C)



(D)

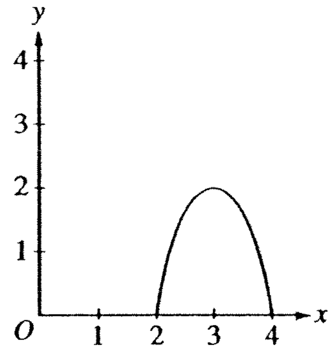


26. $\dfrac{d}{dx}\displaystyle\int_0^x \cos(2\pi u)\,du$ is

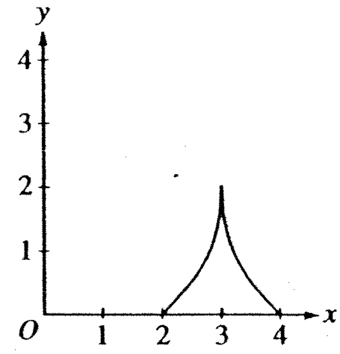
(A) 0 (B) $\frac{1}{2\pi}\sin x$ (C) $\frac{1}{2\pi}\cos(2\pi x)$ (D) $\cos(2\pi x)$ (E) $2\pi\cos(2\pi x)$

27 On the closed interval [2, 4], which of the following could be the graph of a function f with the property that $\frac{1}{4-2}\int_2^4 f(t)\,dt = 1$?

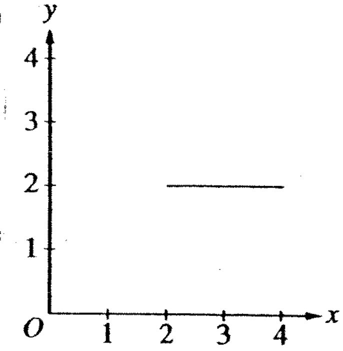
(A)



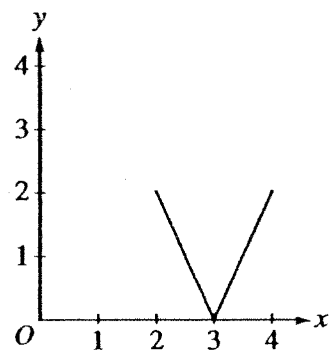
(B)



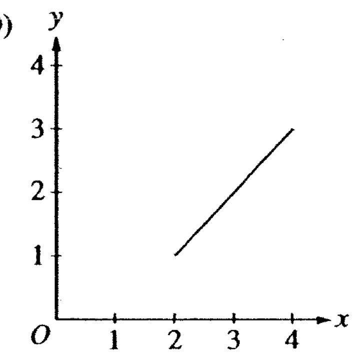
(E)

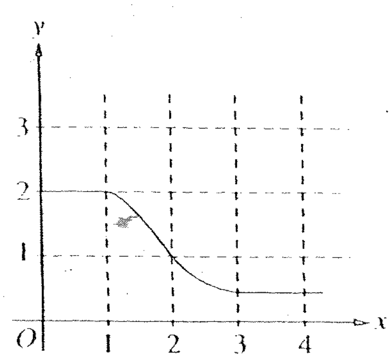


(C)



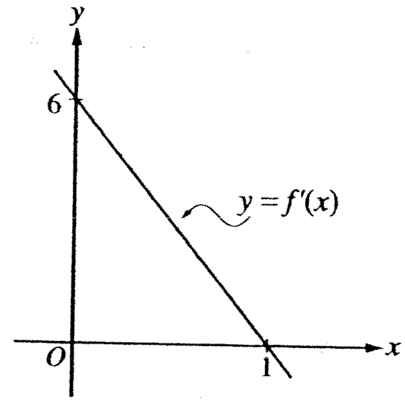
(D)





28. The graph of f is shown in the figure above. If $\int_1^3 f(x)\,dx = 2.3$ and $F'(x) = f(x)$, then $F(3) - F(0) =$

(A) 0.3 (B) 1.3 (C) 3.3 (D) 4.3 (E) 5.3



29. The graph of f', the derivative of f, is the line shown in the figure above. If $f(0) = 5$, then $f(1) =$

(A) 0 (B) 3 (C) 6 (D) 8 (E) 11

30. If f is a continuous function and if $F'(x) = f(x)$ for all real numbers x, then $\int_1^3 f(2x)\,dx =$

(A) $2F(3) - 2F(1)$

(B) $\frac{1}{2}F(3) - \frac{1}{2}F(1)$

(C) $2F(6) - 2F(2)$

(D) $F(6) - F(2)$

(E) $\frac{1}{2}F(6) - \frac{1}{2}F(2)$

Differential Equations

1. If $\frac{dy}{dt} = ky$ and k is a nonzero constant, then y could be

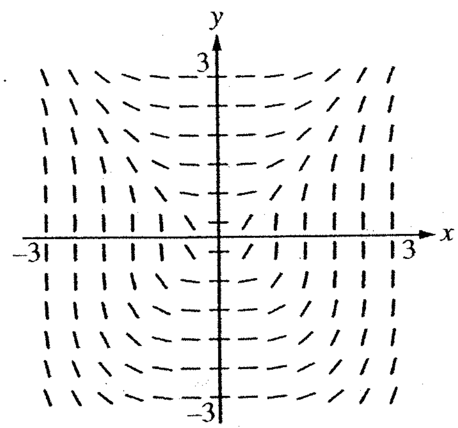
(A) $2e^{kty}$ (B) $2e^{kt}$ (C) $e^{kt} + 3$ (D) $kty + 5$ (E) $\frac{1}{2}ky^2 + \frac{1}{2}$

2. A puppy weighs 2.0 pounds at birth and 3.5 pounds two months later. If the weight of the puppy during its first 6 months is increasing at a rate proportional to its weight, then how much will the puppy weigh when it is 3 months old?

(A) 4.2 pounds (B) 4.6 pounds (C) 4.8 pounds (D) 5.6 pounds (E) 6.5 pounds

3. If $\frac{dy}{dx} = xy^2$ and $y = -\frac{1}{3}$ when $x = 2$, what is y when $x = 4$?

(A) $-\frac{1}{3}$ (B) $-\frac{1}{5}$ (C) $-\frac{1}{9}$ (D) $\frac{1}{3}$ (E) $\frac{1}{9}$



4. Shown above is a slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = \frac{x}{y}$ (B) $\frac{dy}{dx} = \frac{x^2}{y^2}$ (C) $\frac{dy}{dx} = \frac{x^3}{y}$ (D) $\frac{dy}{dx} = \frac{x^2}{y}$ (E) $\frac{dy}{dx} = \frac{x^3}{y^2}$

---

5. The acceleration of a particle moving along the x-axis is $a(t) = 12t - 10$.
At $t = 0$, the velocity is 3.
At $t = 1$, the position is $x = 4$.
Find the position at $t = 2$.

(A) 2 (B) 4 (C) 5 (D) 6 (E) 7

6. Population y grows according to the equation $\frac{dy}{dt} = ky$, where k is a constant and t is measured in years. If the population doubles every 10 years, then the value of k is

(A) 0.069 (B) 0.200 (C) 0.301 (D) 3.322 (E) 5.000

7. At time $t \geq 0$, the acceleration of a particle moving on the x-axis is $a(t) = t + \sin t$. At $t = 0$, the velocity of the particle is -2. For what value of t will the velocity of the particle be zero?

(A) 1.02 (B) 1.48 (C) 1.85 (D) 2.81 (E) 3.14

8. If the second derivative of f is given by $f''(x) = 2x - \cos x$, which of the following could be $f(x)$?

(A) $\frac{x^3}{3} + \cos x - x + 1$

(B) $\frac{x^3}{3} - \cos x - x + 1$

(C) $x^3 + \cos x - x + 1$

(D) $x^2 - \sin x + 1$

(E) $x^2 + \sin x + 1$

---

9. If $\frac{dy}{dx} = 2y^2$ and if $y = -1$ when $x = 1$, then when $x = 2$, $y =$

(A) $-\frac{2}{3}$ (B) $-\frac{1}{3}$ (C) 0 (D) $\frac{1}{3}$ (E) $\frac{2}{3}$

---

10. Suppose air is pumped into a balloon at a rate given by $r(t) = \frac{(\ln t)^2}{t}$ ft^3/sec for $t \geq 1$ sec. If the volume of the balloon is 1.3 ft^3 at $t = 1$ sec, what is the volume of the balloon at $t = 5$ sec?

(A) 2.7 ft^3 (B) 3.0 ft^3 (C) 3.3 ft^3 (D) 3.6 ft^3 (E) 3.9 ft^3

---

11. The acceleration of a particle moving along the x-axis at time t is given by $a(t) = 6t - 2$. If the velocity is 25 when $t = 3$ and the position is 10 when $t = 1$, then the position $x(t) =$

(A) $9t^2 + 1$

(B) $3t^2 - 2t + 4$

(C) $t^3 - t^2 + 4t + 6$

(D) $t^3 - t^2 + 9t - 20$

(E) $36t^3 - 4t^2 - 77t + 55$

---

12. **The rate of change of the volume, V, of water in a tank with respect to time, t, is directly proportional to the square root of the volume. Which of the following is a differential equation that describes this relationship?**

(A) $V(t) = k\sqrt{t}$

(B) $V(t) = k\sqrt{V}$

(C) $\frac{dV}{dt} = k\sqrt{t}$

(D) $\frac{dV}{dt} = \frac{k}{\sqrt{V}}$

(E) $\frac{dV}{dt} = k\sqrt{V}$

The weight in pounds of a certain bear cub t months after birth is given by $w(t)$. If $w(2) = 36$, $w(7) = 84$, and $\frac{dw}{dt}$ was proportional to the cub's weight for the first 15 months of his life, how much did the cub weigh when he was 11 months old?

(A) 125 pounds (B) 135 pounds (C) 145 pounds (D) 155 pounds (E) 165 pounds

AP Calculus AB–6

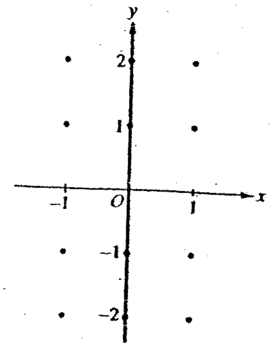
2005

Consider the differential equation $\frac{dy}{dx} = -\frac{2x}{y}$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.
(Note: Use the axes provided in the pink test booklet.)

(b) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(1) = -1$. Write an equation for the line tangent to the graph of f at $(1, -1)$ and use it to approximate $f(1.1)$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(1) = -1$.



1989 AB 6

Oil is being pumped continuously from a certain oil well at a rate proportional to the amount of oil left in the well; that is, $\frac{dy}{dt} = ky$, where y is the amount of oil left in the well at any time t. Initially there were 1,000,000 gallons of oil in the well, and 6 years later there were 500,000 gallons remaining. It will no longer be profitable to pump oil when there are fewer that 50,000 gallons remaining.

(a) Write an equation for y, the amount of oil remaining in the well at time t.
(b) At what rate is the amount of oil in the well decreasing when there are 600,000 gallons of oil remaining?
(c) In order not to lose money, at what time t should oil no longer be pumped from the well?

Let $P(t)$ represent the number of wolves in a population at time t years, when $t \geq 0$. The population $P(t)$ is increasing at a rate directly proportional to $800 - P(t)$, where the constant of proportionality is k.

(a) If $P(0) = 500$, find $P(t)$ in terms of t and k.

(b) If $P(2) = 700$, find k.

(c) Find $\lim_{t \to \infty} P(t)$.

1992 AB 6

At time t, $t \geq 0$, the volume of a sphere is increasing at a rate proportional to the reciprocal of its radius.
At $t = 0$, the radius of the sphere is 1 and at $t = 15$, the radius is 2. (The volume V of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.)

Integration Applications

1. A particle moves along the x-axis so that at any time $t > 0$, its acceleration is given by $a(t) = \ln\left(1 + 2^t\right)$. If the velocity of the particle is 2 at time $t = 1$, then the velocity of the particle at time $t = 2$ is

(A) 0.462 (B) 1.609 (C) 2.555 (D) 2.886 (E) 3.346

2. The rate of change of the altitude of a hot-air balloon is given by $r(t) = t^3 - 4t^2 + 6$ for $0 \le t \le 8$. Which of the following expressions gives the change in altitude of the balloon during the time the altitude is decreasing?

(A) $\int_{1.572}^{3.514} r(t)\,dt$

(B) $\int_{0}^{8} r(t)\,dt$

(C) $\int_{0}^{2.667} r(t)\,dt$

(D) $\int_{1.572}^{3.514} r'(t)\,dt$

(E) $\int_{0}^{2.667} r'(t)\,dt$

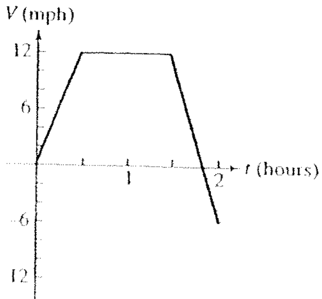
3. A particle with velocity at any time t given by $v(t) = e^t$ moves in a straight line. How far does the particle move from $t = 0$ to $t = 2$?

(A) $e^2 - 1$ (B) $e - 1$ (C) $2e$ (D) e^2 (E) $\frac{e^3}{3}$

4. $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\cos\theta}{\sqrt{1 + \sin\theta}}\,d\theta =$

(A) $-2(\sqrt{2} - 1)$ (B) $-2\sqrt{2}$ (C) $2\sqrt{2}$ (D) $2(\sqrt{2} - 1)$ (E) $2(\sqrt{2} + 1)$

5.



A bicyclist rides along a straight road starting from home at $t = 0$. The graph above shows the bicyclist's velocity as a function of t. How far from home is the bicyclist after 2 hours?

(A) 13 miles (B) 16.5 miles (C) 17.5 miles (D) 18 miles (E) 20 miles

6. The area of the region enclosed by the curve $y = \frac{1}{x-1}$, the x-axis, and the lines $x = 3$ and $x = 4$ is

(A) $\frac{5}{36}$ (B) $\ln\frac{2}{3}$ (C) $\ln\frac{4}{3}$ (D) $\ln\frac{3}{2}$ (E) $\ln 6$

---

7. What is the average value of $y = x^2\sqrt{x^3+1}$ on the interval $[0, 2]$?

(A) $\frac{26}{9}$ (B) $\frac{52}{9}$ (C) $\frac{26}{3}$ (D) $\frac{52}{3}$ (E) 24

8. $\int_0^{\frac{\pi}{4}} \frac{e^{\tan x}}{\cos^2 x}\, dx$ is

(A) 0 (B) 1 (C) $e-1$ (D) e (E) $e+1$

9. $\int_2^3 \frac{x}{x^2+1}\, dx =$

(A) $\frac{1}{2}\ln\frac{3}{2}$ (B) $\frac{1}{2}\ln 2$ (C) $\ln 2$ (D) $2\ln 2$ (E) $\frac{1}{2}\ln 5$

10. Let R be the region in the first quadrant enclosed by the graph of $y = (x+1)^{\frac{1}{3}}$, the line $x = 7$, the x-axis, and the y-axis. The volume of the solid generated when R is revolved about the y-axis is given by

(A) $\pi\int_0^7 (x+1)^{\frac{2}{3}}\, dx$ (B) $2\pi\int_0^7 x(x+1)^{\frac{1}{3}}\, dx$ (C) $\pi\int_0^2 (x+1)^{\frac{2}{3}}\, dx$

(D) $2\pi\int_0^2 x(x+1)^{\frac{1}{3}}\, dx$ (E) $\pi\int_0^7 (y^3-1)^2\, dy$

11. The area of the region enclosed by the graphs of $y = x$ and $y = x^2 - 3x + 3$ is

(A) $\frac{2}{3}$ (B) 1 (C) $\frac{4}{3}$ (D) 2 (E) $\frac{14}{3}$

12. If $0 \le k < \frac{\pi}{2}$ and the area under the curve $y = \cos x$ from $x = k$ to $x = \frac{\pi}{2}$ is 0.1, then $k =$

(A) 1.471 (B) 1.414 (C) 1.277 (D) 1.120 (E) 0.436

13. If the region enclosed by the y-axis, the line $y = 2$, and the curve $y = \sqrt{x}$ is revolved about the y-axis, the volume of the solid generated is

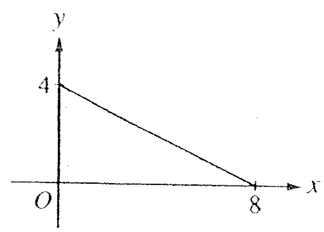
(A) $\frac{32\pi}{5}$ (B) $\frac{16\pi}{3}$ (C) $\frac{16\pi}{5}$ (D) $\frac{8\pi}{3}$ (E) π

14. A pizza, heated to a temperature of 350 degrees Fahrenheit (°F), is taken out of an oven and placed in a 75°F room at time $t = 0$ minutes. The temperature of the pizza is changing at a rate of $-110e^{-0.4t}$ degrees Fahrenheit per minute. To the nearest degree, what is the temperature of the pizza at time $t = 5$ minutes?

(A) 112°F (B) 119°F (C) 147°F (D) 238°F (E) 335°F

15. $\int_0^{\sqrt{3}} \frac{dx}{\sqrt{4 - x^2}} =$

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{1}{2}\ln 2$ (E) $-\ln 2$



16. The base of a solid is a region in the first quadrant bounded by the x-axis, the y-axis, and the line $x + 2y = 8$, as shown in the figure above. If cross sections of the solid perpendicular to the x-axis are semicircles, what is the volume of the solid?

(A) 12.566 (B) 14.661 (C) 16.755 (D) 67.021 (E) 134.041

17. What is the average value of y for the part of the curve $y = 3x - x^2$ which is in the first quadrant?

(A) -6 (B) -2 (C) $\frac{3}{2}$ (D) $\frac{9}{4}$ (E) $\frac{9}{2}$

18. $\int_1^4 |x - 3|\, dx =$

(A) $-\frac{3}{2}$ (B) $\frac{3}{2}$ (C) $\frac{5}{2}$ (D) $\frac{9}{2}$ (E) 5

19. The volume of the solid obtained by revolving the region enclosed by the ellipse $x^2 + 9y^2 = 9$ about the x-axis is

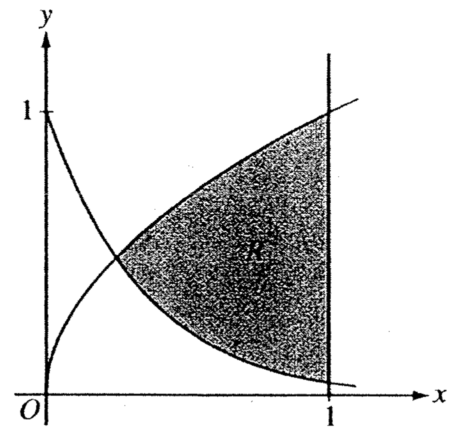
(A) 2π (B) 4π (C) 6π (D) 9π (E) 12π

20. What is the area of the region in the first quadrant enclosed by the graphs of $y = \cos x$, $y = x$, and the y-axis?

(A) 0.127 (B) 0.385 (C) 0.400 (D) 0.600 (E) 0.947

21 The base of a solid is the region in the first quadrant bounded by the y-axis, the graph of $y = \tan^{-1} x$, the horizontal line $y = 3$, and the vertical line $x = 1$. For this solid, each cross section perpendicular to the x-axis is a square. What is the volume of the solid?

(A) 2.561 (B) 6.612 (C) 8.046 (D) 8.755 (E) 20.773



1. Let R be the shaded region bounded by the graphs of $y = \sqrt{x}$ and $y = e^{-3x}$ and the vertical line $x = 1$, as shown in the figure above.
   (a) Find the area of R.
   (b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 1$.
   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a rectangle whose height is 5 times the length of its base in region R. Find the volume of this solid.

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2. The temperature outside a house during a 24-hour period is given by

$$F(t) = 80 - 10\cos\left(\frac{\pi t}{12}\right),\quad 0 \le t \le 24,$$

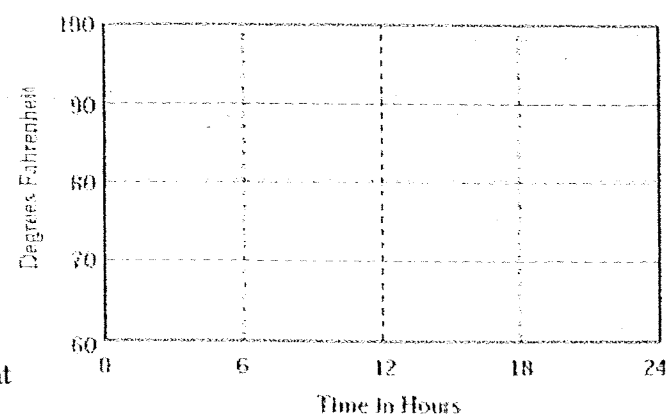
where $F(t)$ is measured in degrees Fahrenheit and t is measured in hours.

(a) Sketch the graph of F on the grid to the right.

(b) Find the average temperature, to the nearest degree Fahrenheit, between $t = 6$ and $t = 14$.

(c) An air conditioner cooled the house whenever the outside temperature was at or above 78 degrees Fahrenheit. For what values of t was the air conditioner cooling the house?

(d) The cost of cooling the house accumulates at the rate of \$0.05 per hour for each degree the outside temperature exceeds 78 degrees Fahrenheit. What was the total cost, to the nearest cent, to cool the house for this 24-hour period?



Click here to view larger figure.