Review of sequences and series

Multiple Choice

1. The common difference in the arithmetic sequence 2, −6, −14, −22, . . . is
   a. −8
   b. −3
   c. −16
   d. 8

2. In the formula for the general term of an arithmetic sequence \( t_n = -7 + (n - 1) \times (-2.5) \), the common difference is
   a. 17.5
   b. −7
   c. −4.5
   d. −2.5

3. The sum of the series \((-5) + (-7) + (-9) + \ldots + (-19)\) is
   a. −96
   b. −304
   c. −192
   d. 26

4. The eighth term in the sequence 3 515 625, 703 125, 140 625, 28 125, . . . is
   a. 9
   b. \(\frac{1}{9}\)
   c. 45
   d. 5

5. Determine the sum of the infinite geometric series \(11 + \frac{11}{3} + \frac{11}{9} + \frac{11}{27} + \ldots\)
   a. 33
   b. \(\frac{33}{4}\)
   c. \(\frac{33}{2}\)
   d. \(\frac{440}{27}\)

Short Answer

For each arithmetic sequence, determine
a) the value of \(t_1\) and \(d\)
b) an explicit formula for the general term
c) \(t_{20}\)

6. −8, −5, −2, 1, . . .

   Determine whether each sequence is geometric, arithmetic, or neither. Justify your answer.

7. \(\frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, 1, . . .\)

For each geometric sequence, determine
a) an explicit formula for the general term
b) \(t_{11}\)

8. \(t_1 = 3, r = 2\)
For each arithmetic series, determine
a) an explicit formula for the general term
b) a formula for the general sum
c) \( t_{12} \)
d) \( S_n \)

9. \[ 2 + 4 + 6 + \cdots + 48 \]

Determine the sum of each arithmetic series.

10. \[ 4k + 11k + 18k + \cdots + 74k \]

Problem

11. In a lottery to join a golf club, the first person drawn from the names must pay $14,000. Each subsequent person drawn pays $250 less than the person before. The last person drawn pays $8000 for a membership.
   a) Write the first four terms of the sequence that represents the cost of a membership.
   b) Determine \( t_1 \) and \( d \) for the sequence.
   c) Determine an explicit formula for the general term.
   d) What will the 10th golfer pay for a membership?
   e) How many golfers will be able to join the club?

12. The value of an antique increases in value at a rate of 2.5% every year. In 2000, the antique was purchased for $5000.
   a) Determine an explicit formula to represent the value of the antique since the year 2000.
   b) Use your formula to write the first three terms of the sequence.
   c) What was the value of the antique in 2008?
   d) In which year will the value of the antique be $11,866?
Review of sequences and series
Answer Section

MULTIPLE CHOICE

1. ANS: A   PTS: 1   DIF: Easy   OBJ: Section 1.1
   NAT: RF 9   TOP: Arithmetic Sequences   KEY: common difference

2. ANS: D   PTS: 1   DIF: Easy   OBJ: Section 1.1
   NAT: RF 9   TOP: Arithmetic Sequences   KEY: common difference | general term

3. ANS: A   PTS: 1   DIF: Average   OBJ: Section 1.2
   NAT: RF 9   TOP: Arithmetic Series   KEY: sum | number of terms | arithmetic series

4. ANS: C   PTS: 1   DIF: Average   OBJ: Section 1.3
   NAT: RF 10   TOP: Geometric Sequences   KEY: terms | geometric sequence

5. ANS: C   PTS: 1   DIF: Average   OBJ: Section 1.5
   NAT: RF 10   TOP: Infinite Geometric Series   KEY: sum | infinite geometric series

SHORT ANSWER

6. ANS:
   a) $t_1 = -8, d = 3$
   b) $t_n = -8 + (n - 1)(3)$
      $= -8 + 3n - 3$
      $= 3n - 11$
   e) $t_{20} = 3(20) - 11$
      $= 60 - 11$
      $= 49$

   PTS: 1   DIF: Easy   OBJ: Section 1.1   NAT: RF 9
   TOP: Arithmetic Sequences   KEY: terms | explicit formula | arithmetic sequence

7. ANS:
   Since the sequence has neither a common difference nor a common ratio, it is neither geometric nor arithmetic.

   PTS: 1   DIF: Average   OBJ: Section 1.1 | Section 1.3
   NAT: RF 9   TOP: Arithmetic Sequences | Geometric Sequences
   KEY: type of sequence
8. ANS:
   a) \( t_n = 3(2)^{n-1} \)
   b) \( t_{11} = 3(2)^{11-1} \)
      \[ = 3(2)^{10} \]
      \[ = 3072 \]

PTS: 1  DIF: Easy  OBJ: Section 1.3  NAT: RF 9
TOP: Geometric Sequences  KEY: explicit formula | terms | geometric sequence

9. ANS:
   a) \( t_n = t_1 + (n - 1)d \)
      \[ = 2 + (n - 1)(2) \]
      \[ = 2n \]
   b) \( S_n = \frac{n}{2} \left[ 2t_1 + (n - 1)d \right] \)
      \[ = \frac{n}{2} \left[ 2(2) + (n - 1)(2) \right] \]
      \[ = \frac{n}{2} (2n + 2) \]
      \[ = n^2 + n \]
   c) \( t_{12} = 2(12) \)
      \[ = 24 \]
   d) Since \( t_n = 48 \),
      \[ 48 = 2n \]
      \[ n = 24 \]
      \[ S_{24} = 24^2 + 24 \]
      \[ = 600 \]

PTS: 1  DIF: Average  OBJ: Section 1.2  NAT: RF 9
TOP: Arithmetic Series  KEY: explicit formula | sum | terms | arithmetic series
10. ANS: 
\[ t_1 = 4k, \ d = 7k \]
\[ t_n = t_1 + (n - 1)d \]
\[ = 4k + (n - 1)(7k) \]
\[ = 4k + 7kn - 7k \]
\[ = 7kn - 3k \]
\[ t_n = 74k \]
\[ 74k = 7kn - 3k \]
\[ 77k = 7kn \]
\[ n = 11 \]
\[ S_n = \frac{n}{2}[2t_1 + (n - 1)d] \]
\[ S_{11} = \frac{11}{2}[2(4k) + (10)(7k)] \]
\[ = \frac{11}{2}(8k + 70k) \]
\[ = 429k \]

PTS: 1 DIF: Average OBJ: Section 1.2 NAT: RF 9
TOP: Arithmetic Series KEY: sum | arithmetic series
PROBLEM

11. ANS:
   a) 14 000, 13 750, 13 500, 13 250
   b) $t_i = 14 000, d = -250$
   c) $t_n = t_1 + (n - 1)d$
      
      
      
      
      
      
      
      
      
      
      
      
      
      
      
      
      
      
      
      
      
      
      
      
      
      
      
      
      
      
      
      
      
      Therefore, the 10th golfer will pay $11 750.
   e) The last person pays $8000, so $t_n = 8000$.
      
      
      
      
      
      
      

   Therefore, 25 golfers will be able to join the club.

PTS: 1 DIF: Average OBJ: Section 1.1 NAT: RF 9

12. ANS:
   a) At a rate of increase of 2.5% per year, $r = 1.025$. The initial value of the antique in 2000 is $5000, so $t_0 = 5000$.
      
      
      
      
      
      
      
      
      
      
      
      
      
      
      
      
      
      
      
      
      
      
      
      
      
      
      
      
      
      
      
      Using systematic trial, $(1.025)^{35} = 2.3732$. So, $n = 35$. In 2035, the antique will be worth approximately $11 866.

PTS: 1 DIF: Average OBJ: Section 1.3 NAT: RF 9